Abstract

Bender-element (BE) tests were conducted on clay-sand mixtures to investigate the variation of small strain-shear modulus \((G_{\text{max}})\) with the sand content and the physical characteristics (size, shape) of the sand grains in the mixtures. Three different gradations (0.6–0.3 mm, 1.0–0.6 mm and 2.0–1.0 mm) of sands having distinct shapes (rounded, angular) were added to a low-plasticity clay with mixture ratios of 0% (clean clay), 10%, 20%, 30%, 40%, and 50%. For the purposes of performing a correlation analysis, unconfined compression (UC) tests were also carried out on the same specimens. The tests indicated that both the \(G_{\text{max}}\) and unconfined compressive strength \((q_u)\) values of the specimens with angular sand grains were measured to be lower than those with rounded sand grains, for all sizes and percentages. As the percentage of sand in the mixture increases, the \(G_{\text{max}}\) values increase, while the \(q_u\) values decrease. The results further suggested that the \(G_{\text{max}}\) values decrease as the \(q_u\) values decreases as the size of the sand grains reduces.

1 INTRODUCTION

Most of the experimental studies for determining the engineering characteristics of soils focused on the behavior of clean soil samples. However, site investigations showed that most of the soil types contain both cohesive and cohesionless grains with various chemical and physical properties [1]. The governing role of either cohesive or cohesionless grain matrices on the overall behavior of the sample can be expected to change based on the properties of both materials. The interaction between cohesionless and cohesive grain matrices can affect the overall behavior of such mixed soils subjected to various dynamic loadings as well as monotonic loadings. Actually, it is of great importance to identify the dynamic behavior of such soils in order to make an accurate stability analysis of any systems subjected to cyclic loads, for example, those resulting from earthquakes, machine foundation, sea waves, wind, and traffic loads [2,3]. Many problems in such loadings are dominated by wave propagation effects where only low levels of strain are induced in the soil. The shear modulus and the damping ratio are the most significant soil properties influencing such small strain behavior. Small strains, typically shear strains of less than 0.001%, do not cause an important nonlinear stress-strain response in the soil. Hence, an equivalent linear model can be assumed. The most significant stiffness parameter at this level is the small strain shear modulus, \(G_{\text{max}}\), which is a parameter required for advanced soil modeling as well as solving soil dynamic problems. A common way of determining...
the small strain shear modulus is to measure the shear wave velocity and then compute $G_{\text{max}}$ as follows [4,5]:

$$G_{\text{max}} = \rho v_s^2 \text{ with } v_s = \frac{L}{t}$$  \hspace{1cm} (1)

where $\rho$ is the density of the soil, $v_s$ is the shear wave velocity, $L$ is the wave path length between the tips of source and receiver elements, and $t$ is the shear wave travel time. In order to determine the small strain shear modulus of soils, one of the commonly used laboratory tests is the Bender Element (BE), which consists of two thin plates of piezoelectric material bonded together, with two conductive outer layers and a metal shim at the center [6]. In spite of the difficulties in determining the shear wave’s arrival time [7], a soil sample can be tested subsequently for other soil characteristics, because the BE tests do not disturb the soil samples, thus facilitating a comparison with other results [8-11]. As characterizing the behaviour of specimens under dynamic loading becomes much more complex and expensive than that of specimens subjected to monotonic loading, only a few researchers have been carried out on correlating a dynamic test with a static test so far [12-13]. Therefore, there is a real need for an in-depth investigation of a comparison between dynamic tests and static tests. Actually, both studies by Consoli et al. [12] and Flores et al. [13] suggest that the unconfined compressive strength ($q_u$) values and $G_{\text{max}}$ defined by the BE can be closely correlated for artificially cemented soils. The unconfined compression (UC) test is by far the most popular technique for soil shear testing as it is one of the cheapest and fastest techniques for measuring shear strength. The technique is employed primarily for cohesive soils recovered from thin-walled sampling tubes [14].

It has long been understood that the physical properties (e.g., size, shape) of sand grains have a significant influence on the engineering properties of a soil matrix [15-23]. Terzaghi [15] postulated that the compressibility of a cohesionless material is governed by the grain size, uniformity, volume of voids and mica content. The observations, made by Gilboy [16], that any system of analysis or classification of soil that neglects the presence and effect of the shape will be incomplete and erroneous. Holubec and D’Appolinia [24] showed that the results of dynamic penetration tests in sands depend on the grain shape. Cornforth [25] and Holtz and Kovacs [26] demonstrated how the grain shape impacts on the internal friction angle ($\phi$). Cedergen [27] pointed out that the grain shape affects the permeability. Holubec and D’Appolinia [24], Wadell [28], Krumbein [29], Powers [30], Youd [31], and Cho et al. [32] have introduced detailed explanations relating to grain shape. Two independent properties are typically employed to describe the shape of a soil grain: (i) Roundness is a measure of the extent to which the edges and corners of a grain have been rounded; (ii) Sphericity (form) describes the overall shape of a grain (it is a measure of the extent to which a grain approaches a sphere in shape). Wadell [28] proposed a simplified sphericity ($S$) parameter, $D_{\text{max-insc}}/D_{\text{min-circ}}$, where $D_{\text{max-insc}}$ is the diameter of a maximum inscribed circle and $D_{\text{min-circ}}$ is the diameter of a minimum sphere circumscribing a gravel grain. Wadell [28] defined roundness ($R$) as $D_{\text{ave}}/D_{\text{max-insc}}$.

$$D_{\text{max-insc}} \quad D_{\text{min-circ}}$$

Figure 1. Graphical representation of roundness, $R$ (redrawn from Muszynski and Vitton, 2012).

$$D_{\text{max-insc}} \quad D_{\text{min-circ}}$$

Figure 2. Graphical representation of sphericity, $S$ (redrawn from Muszynski and Vitton, 2012).

Figure 3. Comparison chart (Santamarina and Cho, 2004).
where $D_{i\text{ave}}$ is the average diameter of the inscribed circle for each corner of the grain. Figures 1-3 define $R$, $S$ and a chart for comparing them to determine grain shape, respectively [29, 30].

The soils are usually obtained from different sites having a wide range of cohesive and cohesionless grains. The main focus of this investigation is defining the small strain shear modulus ($G_{\text{max}}$) of clay-sand mixtures whose properties are intermediate between those of clays and sands. $G_{\text{max}}$ is a significant factor representing the small strain response of soils under seismic load, and an important parameter in the design of foundations where only a small deformation takes place. In the present study, small strain shear modulus measurements ($G_{\text{max}}$) of various compacted clay-sand mixtures were performed at optimum water content ($w_{\text{opt}}$) conditions. The tests were carried out using a testing apparatus that allows Bender Element (BE) and Unconfined Compression (UC) tests to be conducted on an identical specimen, and in this way a more reliable comparison will be obtained. Tests on the mixtures of clay and sands with different contents and physical properties were performed in an attempt to explain the differences in the behaviors of the shear waves between the test specimens. The objective of this study was partly scientific curiosity, but also to judge the usefulness of UC tests in predicting the $G_{\text{max}}$ of clay-sand mixtures. Accordingly, this study will focus on two important aspects of measuring the $G_{\text{max}}$ of clay-sand mixtures: (i) identifying the specific influence of quantity, size, and shape of sand grains in the mixtures, where factors such as density were maintained constant for each specimen, (ii) assessing the testing results deduced from the BE and UC tests on the same specimens.

2 EXPERIMENTAL STUDY

2.1 Materials

The materials used in the tests to produce the mixtures were clay, Crushed Stone Sand, Narli Sand, and de-aired water.

The clay used in the experimental studies was quarried from the Gaziantep University Campus. Its plastic limit and liquid limit values are 23, and 48, respectively [33]. The specific gravity ($G_s$) of the clay grains was found to be 2.61. Based on the Unified Soil Classification System (USCS), the clay was classified as 'low plasticity clay' (CL). Figure 4 shows the scanning electron micrograph (SEM) pictures of the clay used during the experimental investigations. Narli Sand (NS), representing a type of rounded sand, was quarried in and around Narli, Kahramanmaraş in southern-central Turkey; it is widely consumed in earthworks in certain regions of Turkey. As can be seen clearly from Figure 4, Narli Sand grains have a rounded, whereas the Crushed Stone Sand grains have angular, shape of grains. The specific gravity of the grains was found to be 2.65 for Narli Sand, and 2.68 for Crushed Stone Sand. Three different gradations of the sands falling between 0.6 mm and 0.3 mm, 1.0 mm and 0.6 mm, and 2.0 mm and 1.0 mm were artificially selected to provide uniform specimens for visual classification purposes (Figure 5), and then were added to the CL type clay at mixture ratios of 0%, 10%, 20%, 30%, 40%, and 50%. The roundness ($R$) estimates for the angular sand (CSS) and rounded sand (NS) were obtained as 0.16 and 0.43, while the sphericity ($S$) estimates were found to be 0.55 and 0.67, respectively. Further observations of the grain size and shape analysis
2.2 Testing apparatus and specimen preparation

Bender-element (BE) tests were conducted on the different clay-sand mixtures to measure the shear wave velocity at various states in order to estimate the small strain shear modulus \( G_{\text{max}} \). The tests were carried out just before applying a uniaxial load to the compacted specimens taken out from a two-part split mould with a 70-mm diameter and a 147-mm height. The shear wave velocity measurements were performed with a pair of bender elements, one of which was installed on the pedestal, and the other one was placed in the top cap. The bender element on the pedestal was used as transmitter, while the other in top cap was used as a receiver to measure the shear wave propagated through a specimen. The software package for data acquisition as well as the BE testing equipment used during the experimental works is a product of GDS Instruments Limited. The \( v_s \) estimates were achieved by employing a sinusoidal wave input with a magnitude of 10 V, and periods of 0.01 mV. The received wave was acquired using a sampling frequency of 15kHz, and a sampling interval of 5 ms, which were selected to provide a received signal with an optimal resolution. From the study by Viggiani and Atkinson [35], the shear wave velocity transmitted through the specimens was estimated using the ‘tip-to-tip’ distance between the bender elements, and then the wave travel time was measured. In order to obtain the arrival time value, the first major peak-to-peak method was employed. The arrival time in this method was defined as the time measured between the peak of the transmitted signal and the first major peak of the received signal.

The unconfined compression (UC) test, which is used for measuring undrained shear strength of cohesive soils because of the simplicity of the test technique, was employed on various clay-sand specimens compacted in a mould with a 70-mm in diameter and a 147-mm height [14]. The performance of the specimens was investigated with the clean clay, and clay with sand at the

![Figure 6. The equipment combining BE and UC tests.](image)
mixture ratios of 10%, 20%, 30%, 40%, and 50% using the dry weight of the specimens. The amount of each constituent in a mixture was determined by employing a compaction test in order to prepare the specimens at the maximum dry unit weight ($\gamma_{\text{drymax}}$) and the optimum moisture content ($w_{\text{opt}}$) values. In this experimental work, the required amount of sand, clay, and water were weighed, and then mixed until a uniform mixture was reached. The specimens were kept within plastic bags for a period of 24 hours to avoid the loss of moisture, and to obtain a homogeneous paste. The mixture to be tested was then statically compacted into a two-part cylindrical mould (70mm × 147mm) in three layers. Then, the specimens were taken out of the mould, trimmed out, and placed on to the UC testing equipment in order to estimate the shear wave velocity first, and then the $q_u$ value for each specimen. The rate of strain employed in the UC tests was 0.7 mm/min. Figure 6 presents the equipment used for the experimental determination of both the $G_{\text{max}}$ and $q_u$ values.

3 RESULTS AND DISCUSSION

Table 3 gives a summary of the UC testing results reported here. From the tests with both clay and clay-sand mixtures compacted at $w_{\text{opt}}$ and $\gamma_{\text{drymax}}$, the unconfined compressive strength ($q_u$) values were observed to be significantly affected by the addition of sands. The $q_u$ values decreased as the amount of sand content increased in all size, shape, and content of the sand grains. Similar results were also reported by Stavridakis [36] for an investigation of problematic soils. It is interpreted that the difference in testing results can be attributed to the soil suction as well as quantity and physical characteristics of the sand grains (shape, size) leading to various void ratios. The soil suction could be effective in the shear strength and the deformation characteristics of soils for the highly saturated condition at the unconfined compression test [37-39]. However, it seems to be beyond the scope of this study to provide a detailed discussion on the soil suction, since the amount of water was selected as $w_{\text{opt}}$ for each specimen that creates a very similar range of saturation degree as well as soil suction in all the specimens tested (Table 2). The shape of the sand grains in the mixtures is of great importance to the undrained shear strength behavior of the mixtures tested in the UC tests. As can be seen from Figure 7, the $q_u$ obtained for the clay with CSS grains between 0.6mm and 0.3mm have the minimal values, while the the $q_u$ obtained for the clay with NS grains between 2.0mm and 1.0mm have the maximal values. Actually, the mixtures with angular shaped grains (CSS) at all three gradations exhibited much lower $q_u$ values for all the contents of sands. For instance, the $q_u$ values of the specimens with 30% rounded sand content (NS)

| Gradation (mm) | Sand content (%) | NS | CSS |
|----------------|------------------|------------------|
|                | $\gamma_{\text{drymax}}$ | $w_{\text{opt}}$ (%) | $S_r$ (%) | $\gamma_{\text{drymax}}$ | $w_{\text{opt}}$ (%) | $S_r$ (%) |
| 0.6-0.3        |                  |                  | |
| 10             | 17.12            | 18.4             | 86.1       | 17.12            | 18.4             | 86.1       |
| 20             | 17.41            | 16.8             | 82.6       | 17.24            | 16.3             | 77.8       |
| 30             | 17.83            | 15.8             | 83.6       | 17.65            | 15.3             | 78.2       |
| 40             | 18.25            | 14.6             | 83.2       | 18.02            | 14.2             | 77.3       |
| 50             | 18.59            | 13.5             | 81.9       | 18.41            | 13.1             | 76.3       |
| (1.0-0.6)      |                  |                  | |
| 10             | 17.58            | 16.2             | 81.9       | 17.44            | 15.7             | 77.4       |
| 20             | 18.04            | 15.2             | 83.3       | 17.86            | 14.8             | 78.3       |
| 30             | 18.57            | 14.3             | 86.1       | 18.39            | 13.9             | 80.5       |
| 40             | 18.88            | 13.4             | 85.6       | 18.69            | 13.0             | 79.5       |
| 50             | 19.12            | 12.6             | 84.5       | 18.93            | 12.3             | 78.6       |
| (2.0-1.0)      |                  |                  | |
| 10             | 17.76            | 15.5             | 80.7       | 17.58            | 15.1             | 76.2       |
| 20             | 18.23            | 14.6             | 82.6       | 18.08            | 14.2             | 77.9       |
| 30             | 18.76            | 13.8             | 85.9       | 18.57            | 13.4             | 80.1       |
| 40             | 19.07            | 12.9             | 85.3       | 18.90            | 12.5             | 79.3       |
| 50             | 19.31            | 12.2             | 84.7       | 19.16            | 11.8             | 78.6       |
were found to be 221 kPa, 246 kPa, and 254 kPa, while those with the same amount of angular sand content (CSS) were 206 kPa, 222 kPa, and 237 kPa for the cohesionless materials of 0.6–0.3 mm, 1.0–0.6 mm, and 2.0–1.0 mm, respectively. In the light of the papers by Cabalar et al. [22], Cho et al. [32], Rothenburg and Bathurst [40], Thornton [41], Cabalar [42], it is probably because of the open fabric structure in the specimens with angular sand grains (CSS). Hence, based on the grain fabric taking place in the specimens, the contact points (coordination number) among soil grains decrease, the overall behaviour of the mixture is controlled by the void ratio, \( e \) (Table 3). The sands used during the experimental study have the same classification (SP) although they have different gradation curves. The grain size distribution of the cohesionless materials used in the tests plays a significant role in determining the UC testing results. It was realized that the smaller the size of sand grains the specimens have, the less the \( q_u \) (Figure 7). For example, the specimens with 30% angular sand grains have 206 kPa, 222 kPa, and 237 kPa for the sizes of 0.6–0.3 mm, 1.0–0.6 mm, and 2.0–1.0 mm, respectively. The authors’ interpretation is that the soil grains have much less contact points in the specimens prepared with smaller size sand grains than those with larger sand grains, hence the overall strength behavior of the specimens is mainly governed by the void ratio (\( e \)).

It has long been understood that the void ratio (\( e \)) of a soil matrix is the ratio of voids to the volume of solid grains. The resulting void ratios by the clean angular sand grains are larger than those for clean rounded grains [32, 43]. However, the testing results described here have not been conducted on the clean sands, rather the results have been obtained on the sand-clay mixtures, which are thought of as a composite matrix of finer and coarser soil grains. Therefore, in the light of the numerous investigations [44, 47], analyzing the test results on sand-clay mixtures becomes more versatile if the effects of sand and clay grains are studied separately. This separation appears to be important in describing the use of the intergranular void ratio (\( e_s \)) as an alternative parameter to define the undrained shear-strength behaviour of sand-clay mixtures rather than using traditional void ratio (\( e \)) values [23, 48, 49, 50]. Actually, the intergranular void ratio concept was first proposed by Mitchell [51], and followed by Kenny [52], Lupini et al. [53], and Thevanayagam [54] by assuming that the sand grains can be thought of as the skeleton of soil matrix, and the clay grains occupy the voids between the sand grains, which are defined as the intergranular void ratio (\( e_s \)). The researchers indicated that, based on the amount of clay grains present, the sand grains are in contact with each other and the overall behaviour of the mixture is controlled by the sand grains. On the other hand, when the contact points between the sand grains decrease, the

### Table 3. \( q_u \) values (kPa) for the mixtures tested.

<table>
<thead>
<tr>
<th>Sand content (%)</th>
<th>NS, 0.6-0.3 mm</th>
<th>1.0-0.6 mm</th>
<th>2.0-1.0 mm</th>
<th>CSS, 0.6-0.3 mm</th>
<th>1.0-0.6 mm</th>
<th>2.0-1.0 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>355 0.577</td>
<td>355 0.577</td>
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<td>355 0.577</td>
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<tr>
<td>10</td>
<td>278 0.548</td>
<td>295 0.533</td>
<td>314 0.518</td>
<td>280 0.547</td>
<td>296 0.535</td>
<td>263 0.491</td>
</tr>
<tr>
<td>20</td>
<td>250 0.508</td>
<td>267 0.492</td>
<td>288 0.476</td>
<td>253 0.506</td>
<td>263 0.491</td>
<td>237 0.451</td>
</tr>
<tr>
<td>30</td>
<td>221 0.471</td>
<td>246 0.446</td>
<td>254 0.431</td>
<td>222 0.465</td>
<td>237 0.451</td>
<td>209 0.424</td>
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<tr>
<td>40</td>
<td>190 0.442</td>
<td>209 0.419</td>
<td>223 0.405</td>
<td>197 0.440</td>
<td>209 0.424</td>
<td>176 0.404</td>
</tr>
<tr>
<td>50</td>
<td>163 0.413</td>
<td>174 0.399</td>
<td>187 0.385</td>
<td>163 0.421</td>
<td>176 0.404</td>
<td>151 0.385</td>
</tr>
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</table>

Figure 7. The effect of sand content (%) on the change of \( q_u \) (kPa).
behaviour of the mixture is controlled by the clay grains. From the investigation by Monkul and Ozden [45], the establishment of direct contacts of the sand grains can be initiated when the intergranular void ratio of the mixture becomes equal to the maximum void ratio of the sand grains, i.e., $e_\text{c}=e_{\text{max}}$. The clay content in the case of $e_\text{c}=e_{\text{max}}$ is referred to as ‘transition fines content’. The fact is that the value of $e_\text{c}$ is always higher than the value of $e$ employed in both tests performed in the present study due to the characteristics of the undrained shear strength tests, although the interactions between $e_c$ and $e$ were found to be changed with certain test equipment, including an oedometer, triaxial compression, cyclic triaxial, and resonant column [21,45,55,58]. Hence, the grains of both angular (CSS) and rounded (NS) sands used in the present study are considered as floating in the clay matrix. Then, it was seen that the clay with angular sand grains (CSS) resulted in lower $q_u$ values in UC tests, since the random packing of angular grains resulting void ratios are larger than those of rounded grains (NS). Thus, it was concluded that the less angularity in sand grains the more $e_{\text{c}}$ values obtained in the sand-clay mixtures. Cabalar and Hasan [23] had further stated that (i) mixtures of clay and sand grains with a lower roundness exhibit higher transition fines content ($FC_t$) values and that a (ii) smaller size of sands with clay gives a higher compressibility.

As Hardin [57] stated the void ratio is also influential on the shear modulus ($G_{\text{max}}$). Jamiołkowski et al. [58] supported Hardin [57] considering the statement of void ratio dependence on $G_{\text{max}}$ estimates. As several other researchers have also indicated (Vucetic and Dobry, [59]; Shibuya and Tanaka, [60]; Santagata et al., [61]), the effect of increasing $G_{\text{max}}$ with decreasing void ratio ($e$).

An analysis of the results of the bender-element tests on clay only increased sharply from 23 MPa to 40 MPa by adding 10% rounded sand at a 2.0–1.0mm grain size interval. However, the increment observed for the clay with angular sand at both same amount and grain size interval was found to be slightly lower, i.e., 39 MPa. It was realized that the difference in $G_{\text{max}}$ values of the specimens with same size sand grains is generally less than 5%, which means that the difference observed should not be primarily because of the difference in the grain shape of the sands used in the mixtures. Because they have similar values for coefficient of uniformity ($c_u$), and the coefficient of curvature ($c_c$), besides the mean particle size ($D_{50}$) has an ignorable influence on the elastic modulus [62,63]. Two possible reasons could be the difference in the individual grain contact stiffness due to difference (i) in the mineralogy or the surface roughness, and (ii) the size of the grains. Santamarina and Cascante [64] determined the wave velocity in mildly rusted and rusted steel sphere sand indicated that the surface roughness can decrease the stiffness. However, the sand grains of both angular (CSS) and rounded (NS) sands in the present study are thought to be covered with clay grains, then they do not contact with each other directly. Therefore, it is postulated that the mineralogy or the surface roughness of the sand grains seems to be beyond the scope of this study. To evaluate the influence of the size of sand grains on the $G_{\text{max}}$ values, the results obtained for the same shape of both sand grains with different amounts of clay were compared. For example, the $G_{\text{max}}$ values for the specimen prepared with 40% angular sand between 0.6 mm and 0.3 mm, 1.0 mm and 0.6 mm, and 2.0 mm and 1.0 mm are 37 MPa, 40 MPa, and 42 MPa, respectively. Actually, the $G_{\text{max}}$ values for the clay with smaller grains were found to be always lower than those with larger sand grains. It was seen that the difference in $G_{\text{max}}$ values of the 0.6–0.3mm and 1.0–0.6mm specimens with the same shape sand grains is generally more than 5%, and those of the 0.6–0.3mm and 2.0–1.0mm is more than 10%, which means that the grain size of the sands used in the mixtures has the primary importance. Similar to the observations made

Table 4. $G_{\text{max}}$ values (MPa) for the mixtures tested.

<table>
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<tr>
<th>Sand content (%)</th>
<th>NS</th>
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<td>1.0-0.6 mm</td>
<td>$e$</td>
<td>2.0-1.0 mm</td>
<td>$e$</td>
<td>0.6-0.3 mm</td>
<td>$e$</td>
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by Hardin and Kalinski [65], that the $G_{\text{max}}$ increases with an increase in the mean effective grain size, $D_{50}$ (Figure 8). In addition, the amount of cohesionless material in the mixtures has an obvious influence on the overall behaviour as well as the $G_{\text{max}}$ of such mixtures [44,45,47,66,67] (Figure 9). The evaluated $G_{\text{max}}$ values for the mixtures have been almost doubled when the clay is mixed with 50% sand addition for any size and shape. The $G_{\text{max}}$ of mixtures shows significant changes for a sand content of 10%, which suggests that the threshold value of the sand content at which the trend changing the $e_{\text{max}}$ of the mixtures with clay content should be at less than 10%. The void ratio values, strongly effective on the shear modulus, were found to be changed from 0.577 to 0.385 by adding 50% rounded sand between 2.0–1.0 mm size. Such a decrease in the void ratio has resulted in an increase in $G_{\text{max}}$ from 23 MPa to 44 MPa. Figure 10 reveals that the $G_{\text{max}}$ decreased with an increase of the void ratio for all sizes and both shapes of grains in clay-sand mixtures. The reason for the decrease in $G_{\text{max}}$ is mainly attributed to the increase in the void ratio, which changes based on the size and shape characteristics of the soil grains [5,68,69]. The packing features of the soil control the void ratio and the fabric, which represents the grains' orientation, and the contact patterns of the grains [69,70,71]. The shear waves propagate through a soil matrix with a maximum influence from the contact network. The variation of the void ratio can change the travel length for the wave. The larger the void ratio is, the more travel length is created. Actually, more travel time would be spent in the contact network of the loose soil matrix.

![Figure 8](image1.png) **Figure 8.** The relationship between $G_{\text{max}}$ (MPa) and $D_{50}$ (mm) for various sands in mixtures.

![Figure 9](image2.png) **Figure 9.** The relationship between $G_{\text{max}}$ (MPa) and sand content (%).

![Figure 10](image3.png) **Figure 10.** The relationship between $G_{\text{max}}$ (MPa) and sand content (%).
sample. Conversely, a dense specimen would create a more stable connection, and thus avoids the micro-rotation of the grains. For example, well graded soils with a wide range of grain sizes tend to have smaller average void ratios, and exhibit larger values of the shear wave velocity ($v_s$). Hence, the present study has pointed out that $G_{max}$ is dependent on the void ratio controlled by the grain contents and the characteristics, including size, shape, gradation, and roughness.

The differences between the $G_{max}$ and $q_u$ values are influenced by different conditions under which the dynamic (BE) and static (UC) tests are carried out. The values of $q_u$ in the UC tests are of several hundred of kPa, while the $G_{max}$ during the BE tests does not exceed the value of 50 MPa (Figures 11-12). The loading during UC tests can produce the micro-cracks, which results in the growth of deformation and consequently in the failure of the specimen. However, the BE tests do not change the structure of the material, which is the biggest advantage of their use. The time of applying stress is also different, it lasts several minutes for the UC tests, while it lasts only several microseconds for the BE tests. The fact is that the purpose of the methods for determining the $q_u$ and $G_{max}$ differs based on the demands in practice. If the requirement for determining the $q_u$ formulated by a long-term loading point of view, such as the problem of stability for building works, mining works etc., the $q_u$ values can be obtained from the UC tests. Conversely, if the loading is short term, such as the blasting works, earthquakes, etc., the determination of $G_{max}$ is required. The measurements of $G_{max}$ can be made relatively sophisticated, which is why an alternative method for its determination could be beneficial. Based on the above discussion, the following derived empirical formulas for estimating the $G_{max}$ from the UC tests is suggested for the mixtures of clay-angular sand ($R^2=0.83$), and clay-rounded sand ($R^2=0.76$), respectively.

![Figure 11. Comparison of $G_{max}$ (kPa) and $q_u$ (kPa) of the NS-clay mixtures.](image1)

![Figure 12. Comparison of $G_{max}$ (kPa) and $q_u$ (kPa) of the CSS-clay mixtures.](image2)

![Figure 13. The $G_{max}$ - $q_u$ correlation with sand content (%).](image3)
\[ G_{\text{max}} = -91q_u + 58689 \]  (2)
\[ G_{\text{max}} = -99q_u + 63277 \]  (3)

where \( G_{\text{max}} \) and \( q_u \) values are to be found in kPa.

When the values of \( q_u \) decreased with an increase of the sand percentages in the clay-sand mixtures, the values of \( G_{\text{max}} \) increased for all sizes and both types of sand. From Figure 13 it can also be observed that the values of \( G_{\text{max}} \) increased with an increase of the sand percentages for all sizes and both types of sand for the clay-sand mixtures.

4 CONCLUSIONS

Dynamic deformation characteristics including the small strain shear modulus (\( G_{\text{max}} \)) are the key parameters for the seismic design and a performance evaluation of structures. In the present study, bender-element (BE) tests were performed on various clay-sand mixtures in order to evaluate the effects of quantity (0%, 10%, 20%, 30%, 40%, and 50% by weight), gradation (0.6–0.3 mm, 1.0–0.6 mm, and 2.0–1.0 mm) and shape (rounded, angular) of sand grains on the small strain shear modulus (\( G_{\text{max}} \)) of the mixtures. The unconfined compression (UC) tests were also conducted on the same specimens in order to make a correlation between the two testing results, which enables a platform for the comparison between static load test results and dynamic test results. Based on the outputs obtained, the following conclusions can be drawn.

i) The \( G_{\text{max}} \) of the clay-sand mixtures nearly doubled from about 23 MPa for clean clay to more than 40 MPa for clay with 50% sand, while the \( q_u \) values of the clean clay were found to be about halfway down (from 355 kPa to about 170 kPa) when it was mixed with 50% sand for all sizes and shapes.

ii) For the same amount and shape of the sand grains mixed with clay, the \( G_{\text{max}} \) values of the mixtures decreased for about 5–12%, while the \( q_u \) values of the same specimens increased at about 5–15% by an increase in the mean effective grain size (\( D_{50} \)) of the sands.

iii) Both the \( G_{\text{max}} \) and \( q_u \) values in the specimens prepared with angular sand grains were found to be less than those with rounded grains in the range of about 5–7%, and 2–3%, respectively.

iv) There was a correlation expressed in the form of an analytical function between the \( G_{\text{max}} \) and \( q_u \)

This suggests that both the \( G_{\text{max}} \) and \( q_u \) of the specimens in the way prepared here primarily depend on the void ratio and the size, while they were slightly affected by the grain shape in the mixtures.

REFERENCES


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